*Journal of Applied Mechanics and Technical Physics, Vol. 50, No. 1, pp. 147–154, 2009*

# DETERMINING THE SHEAR ANGLE, FORCES, AND SIZES OF SHEARING ELEMENTS DURING METAL CUTTING

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UDC 539.374

*A new solution for the shear angle is proposed which is a generalization of the solution Lee–Shaffer solution and allows the determination of the cutting force and the shearing-element size. Merchant's experimental data are processed taking into account the resistance force at the cutting edge, and it is shown that accounting for this force leads to the need to increase the internal friction angle in the calculated dependences in order to match theory with experiment. It is shown that the obtained theoretical results agree well with experimental results.*

**Key words:** *metal cutting, shear plane, shearing element, plasticity, fracture, Mohr–Coulomb yield criterion.*

**Introduction.** Metal cutting is one of the basic machining operations; therefore, finding the regularities and main principles of metal cutting is an important theoretical and practical problem, whose solution is the main content of the science of metal cutting [1–9]. New theoretical and experimental results related to the problem of metal cutting are given in [10–14]. In [12–14], statically admissible stress fields admitting the continuation of the solution to the rigid areas are constructed on the basis of the Tresca–Saint Venant yield criterion.

In the theory of metal cutting, the simplest and most widely used solutions are the Ernst–Merchant [1] and Lee–Shaffer solutions [2], which are based on the model assuming the existence of a single isolated shear plane which coincides with the direction of the maximum shear stress. Assuming that the shear stress on the shear plane depends on the normal stress on it, Merchant [3] generalizes the solution of [1], which agrees with the results of metal cutting experiments. Later experimental studies [4] have not supported the solutions obtained in [1–3]. In addition, most of the papers on metal cutting [5–9] contain critical remarks concerning the interpretation of experimental results and the validity of the theoretical assumptions adopted in [3].

Current theories of metal cutting [1–14] do not provide a complete explanation of available experimental results (see, for example, [4]), except in some particular cases. Nevertheless, all cutting theories deepen the understanding of the cutting mechanism, facilitating the development of new methods for studying and solving the problems of material cutting mechanics. The current status of the problem and a complete review of the literature on this topic are presented in [5–14].

The present paper deals with the generalization of the Lee–Shaffer solution [2], which is based on the Mohr–Coulomb plasticity criterion and is in good agreement with experimental results [4]. Experimental support is given for the use of the Merchant solution for experiments [3, 4] in which the cutting conditions differ only slightly. In addition, the present work eliminates the critical remarks on the results of [3] presented in [5–9].

**1. Orthogonal Cutting.** The main element of any cutting tool is the wedge (Fig. 1a). For simplicity, we consider orthogonal cutting in which the cutter in the shape of a two-sided wedge moves at the right angle to the cutting edge. We denote by $\alpha$ the frontal angle of the cutter, i.e., the angle formed by the working face of the wedge and the vertical direction (see Fig. 1a). The simplest and most widely used cutting models assume that the chip OABC (see Fig.1a) behaves as a solid which is in equilibrium under the action of the forces $\boldsymbol{P}$ and $\boldsymbol{R}$

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Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 50, No. 1, pp. 177–186, January–February, 2009. Original article submitted May 29, 2007; revision submitted December 6, 2007.

 

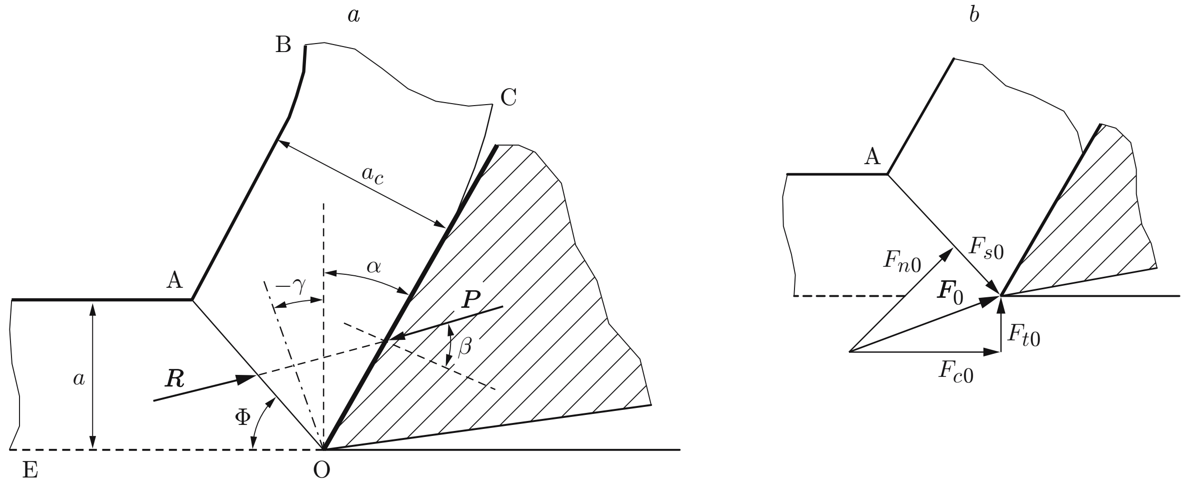


Fig. 1. Diagram of orthogonal cutting with a shear plane: (a) forces acting on the chip from the frontal face of the cutter and the shear plane; (b) projections of the metal resistance force at the cutting edge; the dashed region is the cutter.

transferred, respectively, through the frontal face of the wedge and the shear plane OA which makes an angle $\Phi$ with the direction of motion of the wedge. We assume that the coefficient of the friction force $f$ on the contact surface between the cutter and the chip is determined with sufficient accuracy by the average friction angle $\beta$: $f = \tan\beta$. The force $\boldsymbol{P}$ can be decomposed into two components: in the cutting direction $P_c$ and in a direction perpendicular to it $P_t$. Then, the angle $\beta$ can be determined from the relation $\tan(\beta - \alpha) = P_t/P_c$. If the cutting depth $a$, the frontal angle $\alpha$, and the chip thickness $a_c$ are known, the shear angle can be found from experimental data. The angle $\Phi$ can also be determined theoretically, using the model of a deformation zone with one shear plane and the minimum condition for the cutting work from [1]:

$$\Phi = \pi/4 + (\alpha - \beta)/2. \tag{1}$$

Following Bridgman paper [15] and assuming that the shear stress on the shear plane depends on the normal stress on this plane, Merchant [3] obtained the relation

$$\Phi = \pi/4 + (\alpha - \beta)/2 - \varphi/2, \tag{2}$$

where $\varphi$ is the internal friction angle which shows the dependence of the limiting shear stress on the normal stress for the transition of the metal to the plastic state. A similar formula was obtained in [7].

We consider the Mohr–Coulomb yield condition

$$\max_n (|\tau_n| + \sigma_n \tan\varphi) = C, \tag{3}$$

where $\tau_n$ and $\sigma_n$ are the shear and normal stresses on this plane with the normal $n$ and $C$ is a plastic constant. In [16], condition (3) and the equilibrium equation for the chip element OABC (see Fig. 1) were used to determine the values of the force $P$ and its horizontal $P_c$ and vertical $P_t$ components:

$$P = \frac{Ca\cos\varphi}{\sin\Phi\cos(\Phi + \beta - \alpha + \varphi)}, \qquad P_c = P\cos(\beta - \alpha), \qquad P_t = P\sin(\beta - \alpha). \tag{4}$$

From the minimum condition for the force $P$, we also obtain relation (2) for the shear angle. Using (4), we determine the projections $P_s$ and $P_n$ of the force $\boldsymbol{P}$ onto the directions tangential and normal to the shear plane $s$ and $n$:

$$P_s = P\cos(\Phi + \beta - \alpha), \qquad P_n = -P\sin(\Phi + \beta - \alpha).$$

From these forces, we find the shear and normal stresses on the shear plane:

$$\tau_s = C\cos\varphi\,\frac{\cos(\Phi + \beta - \alpha)}{\cos(\Phi + \beta - \alpha + \varphi)}, \qquad \sigma_n = -C\cos\varphi\,\frac{\sin(\Phi + \beta - \alpha)}{\cos(\Phi + \beta - \alpha + \varphi)}. \tag{5}$$

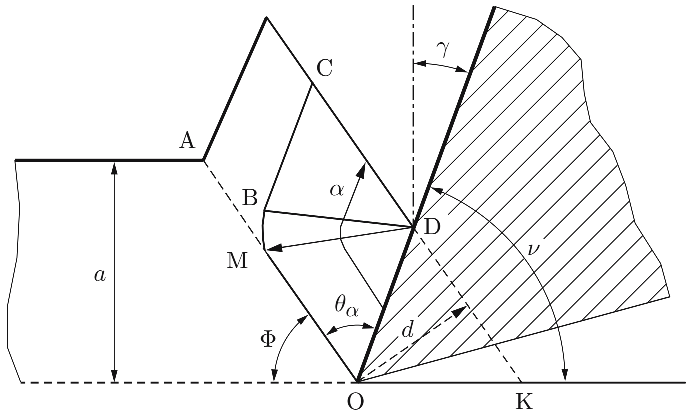


Fig. 2. Rigid plastic model of formation of a shearing element for metal cutting: the dashed region is the cutter without friction.

By analyzing the stress state of the material in the plastic state and taking into account the directions of the slip lines in the chip, Lee and Shaffer [2] obtained the following equation for the shear angle:

$$\Phi = \pi/4 + \alpha - \beta. \tag{6}$$

Papers on the mechanics of metal cutting [5–9] give at least three facts indicating that theories [1–3] are not adequate for a quantitative description of experimental results. First, these theories are inconsistent with experimental results [4]. Second, good agreement between the results of calculation using formula (2) and experimental data [3] is not convincing since, in [3], the experimental results were processed ignoring the resistance force at the cutting edge. Third, formula (2) was derived under the assumption that the shear stress on the shear plane depends on the normal stress, which is unacceptable for plastic metals [5–9]. Using the Mohr–Coulomb criterion to generalize formula (6) and the results of [16], we consider the above-mentioned critical remarks [5–9].

**2. Generalization of the Lee–Shaffer Solution.** Since the force exerted by the frontal face of the wedge on the workpiece is equal in size and opposite in direction to the material cutting resistance transferred through the shear plane, it can be assumed that the direction of the shear plane depends only on the direction of action of the force $\boldsymbol{P}$, and value of this force is determined by the shear angle and the plastic constants of the material. In the presence of friction, the resultant force $\boldsymbol{P}$ on the frontal face of the wedge and the direction of the normal to this face make an angle $\beta$; therefore, in determining the forces acting on the chip, this problem is statically equivalent to the problem in which the frontal face of the wedge is ideally smooth and the frontal angle of the cutter $\gamma = \alpha - \beta$ [16]. For $\alpha > \beta$, the value of the angle $\gamma$ is positive and corresponds to clockwise reading. Conversion to the equivalent problem considerably simplifies the search for new rigid plastic solutions and characteristics of the stress field in the chip on the surface of its contact to the cutter since, in this case, the main stress axes are easily determined. Indeed, since in the equivalent problem, the wedge is smooth, the shear stresses in the chip on the line of contact with the cutter are equal to zero, and, hence, the main axes of the stresses coincide with the tangential and normal directions to the frontal face of the wedge.

Figure 2 shows the rigid plastic model for shearing chip formation proposed in [16]. The characteristics of the first family on the line OD (see Fig. 2) make an angle $\theta_\alpha = \pi/4 + \varphi/2$ with the first principal direction of the stress tensor, which coincides with the line OD, and the third principal direction is perpendicular to this line. The straight line OA, which is a continuation of the $\alpha$-slip line OM and the direction of movement of the cutter make an angle $\Phi = \pi/2 - \theta_\alpha + \gamma$. Substitution of the values for the angles $\theta_\alpha$ and $\gamma$ into this expression yields the following equation for the shear angle:

$$\Phi = \pi/4 + \alpha - \beta - \varphi/2. \tag{7}$$

Relation (7) for $\varphi = 0$ implies the Lee–Shaffer solution (6). Substituting relation (7) into (4) and then into (5), we determine the cutting force and the shear and normal stresses on the shear plane for the generalized solution (7):

$$P^* = \frac{Ca\cos\varphi}{\cos(\pi/4+\varphi/2)\cos(\pi/4-\nu-\varphi/2)}, \qquad \tau_s = C\cos\varphi\tan(\pi/4+\varphi/2),$$
$$P_c = Ca\cos\varphi[1-\tan(\pi/4+\varphi/2)\tan(\pi/4-\nu-\varphi/2)], \qquad \sigma_n = -C\cos\varphi. \tag{8}$$

Here $\nu = \pi/2 - \gamma = \pi/2 - \alpha + \beta$ is the cutting angle of an equivalent wedge with an ideally smooth frontal face. For $\gamma > 0$, the cutting angle is sharp, and for $\gamma < 0$, it is blunt.

Metal chip formation consists of several repeating stages involving the shearing of elements of definite sizes. After the formation of a shear plane, the cutting force $P$ decreases to the minimum value. As the wedge moves a distance $s = \mathrm{OK}$, the force $P$ increases again to the limiting value and a new shear plane is formed. For simplicity, we assume that the shear plane being formed becomes free from stresses since the chip element is completely separated from the chip. In [16], it is assumed that the strength relationship between the shearing element and the chip is retained. For the movement of the wedge in the interval between neighboring shear lines, the rigid-plastic solution [16] with the increasing plastic zone OMBCD holds (see Fig. 2). We formulate the boundary conditions for this problem. Let the $t$ axis coincide with the straight line OD, and let the $n$ direction be perpendicular to OD. Since in the equivalent problem, the wedge with the frontal angle $\gamma$ is ideally smooth, on the straight line OD we have $\sigma_n = -q$ and $\tau_{nt} = 0$. In formulating the boundary conditions on DC, we choose the coordinate system $(n, t)$ in such a manner that the $t$ axis is directed along DC, and $n$ is orthogonal to it. If the formation of the shear plane DC results in the formation of a free surface, i.e., there is no strength relationship between the shear elements, on DC we have $\sigma_n = 0$, $\tau_{nt} = 0$. This nature of shearing chip formation, in which the cut layer of the material is separated by shear lines into separate elements is observed in cutting of brittle metals. In [16], this problem is solved using the method of characteristics separately for $\varphi = 0$ and $\varphi > 0$. The results of this solution for the limiting force $P_d$ directed from the frontal face of the cutter for $\varphi = 0$ are as follows:

$$P_d = q\,|\mathrm{OD}| = C(2+\nu)d/\cos(\nu/2). \tag{9}$$

From (9) and Fig. 2, it follows that, during the movement of the wedge, the distance between the shear lines $d$ increases, and, hence, the load $P_d$ also increases. This occurs until $P_d$ reaches the limiting value $P^*$, determined from (8). Using the condition of formation of a new shear line, from the solution $P_d = P^*$ we determine the quantity $d$:

$$\frac{d}{a} = \frac{\sqrt{2}\,\cos(\nu/2)}{(2+\nu)\cos(\pi/4-\nu/2)}.$$

If $\varphi > 0$, the limiting force $P_d$ is determined from the formula [16]

$$P_d = \frac{Cd\cot\varphi}{\cos(\nu/2-\varphi/2)}\Big(\frac{1+\sin\varphi}{1-\sin\varphi}\exp[(\nu-\varphi)\tan\varphi]-1\Big).$$

Equating this value of the force $P_d$ to the limiting value of the cutting force $P^*$ determined from (8), we find the size of the shearing element

$$\frac{d}{a} = \frac{\sin\varphi\cos(\nu/2-\varphi/2)}{\cos(\pi/4+\varphi/2)\cos(\pi/4-\nu-\varphi/2)\{[(1+\sin\varphi)/(1-\sin\varphi)]\exp[(\nu-\varphi)\tan\varphi]-1\}}. \tag{10}$$

In [16], the size of the shearing element is determined similarly for solution (2). Figure 3 gives curves of the relative size of the shearing element versus the cutting angle calculated by formula (10) using the data of [16]. Figure 4 gives a photomicrograph of a section of the chip root taken during free cutting of 20X steel in water ($\alpha = 0°$, $b = 10$ mm, $a = 0.09$ mm, and $v = 0.7$ m/min) [6]. Assuming that, during cutting, the average friction angle is $\beta = 20$–$30°$, we obtain the cutting angle $\nu = 110$–$120°$, and from relation (10), we have $d/a = 0.6$–$1.6$, which is in qualitative agreement with the experimental results given in Fig. 4.

**3. Comparison of Calculated and Experimental Dependences for the Shear Angle.** We consider the results of experiments with as-received samples of SAE 1112 steel [4]. Figure 5 gives an experimental dependence of the shear angle $\Phi$ on $\beta - \alpha$ for various frontal angles of the cutter at a cutting rates $v = 10.3$ and $27.7$ m/min [4] and the dependence calculated by formula (7) for $\varphi = 20$ and $10°$.

Experimental data on the frontal angles $\alpha = 25$, 20, and 15° for $v = 52.1$ m/min are presented in [4]. As regards cutting conditions, the data obtained are the closest to experimental data [3] for NE 9445 steel. A comparison of calculated dependences (2), (7) and experimental dependences [3, 4] for close cutting conditions leads

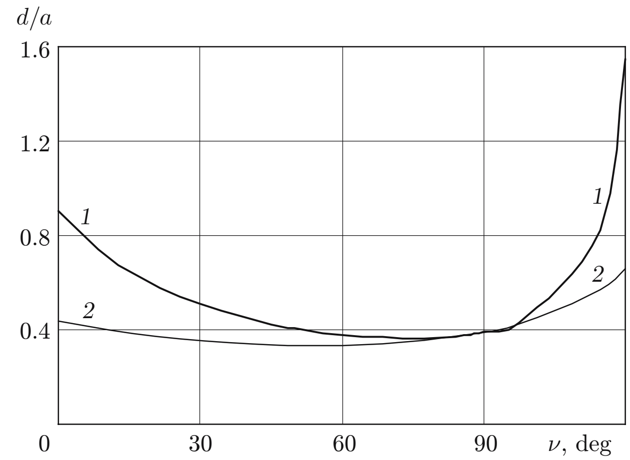



Fig. 3

Fig. 4

Fig. 3. Relative size of the shearing element versus the cutting angle for $\varphi = 20^\circ$: 1) calculation using formula (10); 2) calculation [16].

Fig. 4. Section of the chip root for cutting of 20Kh steel [6].

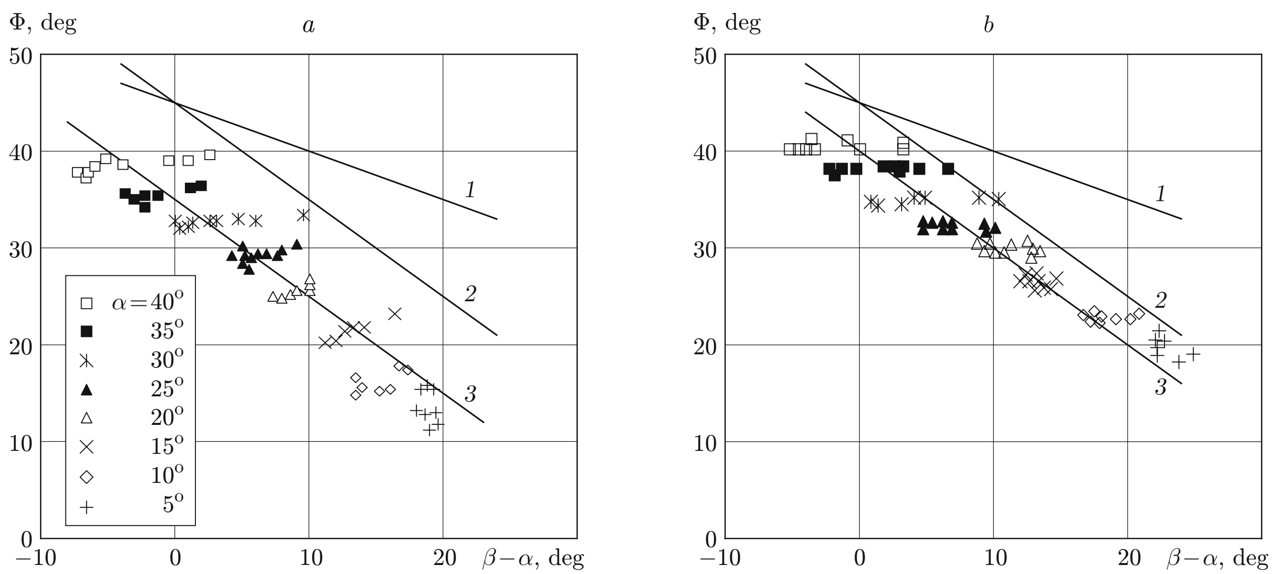


Fig. 5. Shear angle $\Phi$ versus parameter $\beta - \alpha$ for $v = 10.3$ (a) and 27.7 m/min (b); the points refer to the experimental data [4], and curves 1–3 refer to calculations using formulas (1), (6), and (7), respectively for $\varphi = 20^\circ$ (a) and $\varphi = 10^\circ$ (b).

to the following conclusions: the experimental results of [4] agree with the Lee–Shaffer solution [2] only for frontal angles $\alpha = 25$ and $20^\circ$, i.e., with the solution using formula (7) for $\varphi = 0^\circ$; generally, the results of experiments [3, 4] for all frontal angles are in better agreement with solution (2) for $\varphi = 13^\circ$.

From the aforesaid, it follows that with increasing cutting rate, good agreement between the calculated and experimental dependences of the shear angle on $\beta - \alpha$ can be achieved by reducing the internal friction angle in solution (7). For close cutting parameters (cutting rate and the frontal angle of the cutter), the data of experiments [3, 4] agree well with solution (2).

Dependences $\Phi(\beta - \alpha)$ taking into account the passive force (the metal resistance force at the cutting edge) obtained using experimental data [4] are presented in [5]. We consider a method for determining the passive force

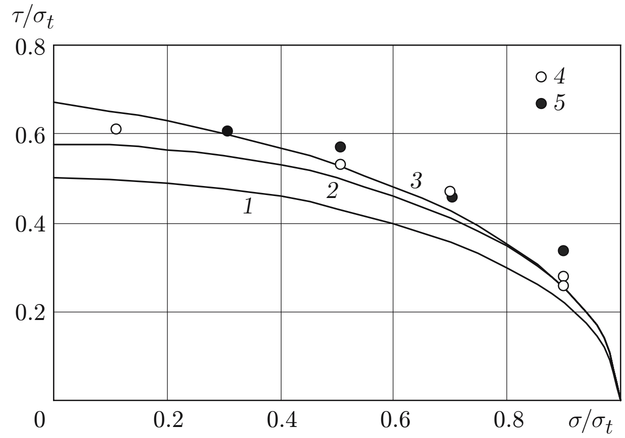


Fig. 6. Stresses $\tau/\sigma_t$ versus $\sigma/\sigma_t$ on the yield surface: curve 1 refer to the calculation using the Tresca–Saint Venant criterion; curve 2 to the calculation using the Huber–Mises criterion, and curve 3 to the calculation using the Mohr–Coulomb criterion for $\varphi = 20°$; points 4 refer to the Taylor and Quinney experimental data of [17] for soft steel and points 5 refer to the experimental data of [12] for carbon-free soft steel.

proposed in [5]. It is generally agreed that, during metal cutting, some part of the force is required to overcome the force of friction between the rear side of the cutter and the surface being processed, and the other part is required to introduce the blunt cutting edge in the material being processed [5, 9]. In this case, the site of application of this force (the rear side of the cutter, the cutting edge or both) is of no significance. It is sufficient to assume that a certain force $\boldsymbol{F}_0$ (see Fig. 1b) independent of the cut thickness [5, 9] acts in the vicinity of the cutting edge. Let $\boldsymbol{F}$ be the resultant force on the cutter, and let $F_c$ and $F_t$ be its horizontal and vertical components (as a rule, these forces are measured in experiments). Using the horizontal component $P_c$ and vertical component $P_t$ of the resultant $\boldsymbol{P}$ on the frontal side of the wedge, the average friction angle can be determined by the formula

$$P_c = F_c - F_{c0}, \qquad P_t = F_t - F_{t0}, \qquad \tan\beta = \frac{P_t + P_c \tan\alpha}{P_c - P_t \tan\alpha}. \tag{11}$$

From an experimental study [9] and relations (8), it follows that the shear stress $\tau_s$ is a constant of the material being processed which can be found from the expression $\tau_s = F_s/A_s$ ($F_s$ is the force acting on a conditional shear area in a plane of section $A_s = bl$, where $b$ is the width of the cut layer and $l = |\mathrm{OA}|$). Following [5], we find the tangential component $F_{s0}$ and normal component $F_{n0}$ of the material resistance force at the cutting edge $F_0$. For this, using experimental data for various frontal angles $\alpha$ and cutting depths $a$, we construct dependences $F_s(l)$ and $F_n(l)$ by the formulas

$$F_s = F_c\cos\Phi - F_t\sin\Phi, \qquad F_n = F_c\sin\Phi - F_t\cos\Phi, \qquad l = a/\sin\Phi.$$

Next, using the least-squares method, we construct the linear dependence $F_s(l)$. As the component of the passive force parallel to the shear plane, we use $F_{s0} = F_s(0)$. The passive-force component $F_{n0} = F_n(0)$ is determined similarly. The projections of the force $\boldsymbol{F}_0$ in the cutting directions $F_{c0}$ and $F_{t0}$ can be calculated by the formulas

$$F_{c0} = F_{s0}\cos\Phi + F_{n0}\sin\Phi, \qquad F_{t0} = F_{n0}\cos\Phi - F_{s0}\sin\Phi. \tag{12}$$

Correcting the experimental data of [4] for the passive force by using the method considered above [5], for a cutting rate $v = 27.7$ m/min we obtain good agreement between experimental dependences of the shear angle and those calculated by formula (7) for $\varphi = 16°$. For $v = 52.1$ m/min, good agreement between experimental data [4] corrected for the passive force and the generalized Lee–Shaffer solution (7) is obtained for $\varphi = 6°$. A comparison of the calculation results and experimental data shows that the solution by formula (7) is in good agreement with experiment. In this case, a decrease in the frontal angle $\alpha$ leads to a decrease in the internal friction angle.

In [3], calculations for NE 9445 steel gave values $F_{s0} = 0.072$ kN and $F_{n0} = 0.26$ kN. Using formulas (12) and (11), we determine the average friction angle $\beta$. Tables 1 and 2 give the average friction angles $\beta$ and $\beta_0$ calculated by formulas (12) for $F_0 = 0$ and $F_0 \neq 0$, respectively. Table 2 gives experimental values of the shear angle $\Phi$ and

TABLE 1

Shear Angle $\Phi$ and Average Friction Angle $\beta$ Determined Ignoring the Passive Force

| Experiment number | $v$, m/min | $a$, mm | $\alpha$, deg | $\Phi$, deg | $F_c$, kN | $F_t$, kN | $f$ | $\beta$, deg |
|---|---|---|---|---|---|---|---|---|
| 1 | 60 | 0.094 | 10 | 17 | 1.646 | 1.214 | 1.05 | 46.4 |
| 2 | 122 | 0.094 | 10 | 19 | 1.601 | 1.259 | 1.12 | 48.2 |
| 3 | 196 | 0.094 | 10 | 21.5 | 1.463 | 0.965 | 0.95 | 43.4 |
| 4 | 361 | 0.094 | 10 | 25 | 1.348 | 0.747 | 0.81 | 39.0 |
| 5 | 122 | 0.094 | −10 | 16.5 | 1.850 | 1.713 | 0.64 | 32.8 |
| 6 | 194 | 0.094 | −10 | 19 | 1.708 | 1.450 | 0.59 | 30.3 |
| 7 | 354 | 0.094 | −10 | 22 | 1.584 | 1.170 | 0.50 | 26.5 |
| 8 | 165 | 0.028 | 10 | 19 | 0.565 | 0.449 | 1.13 | 48.5 |
| 9 | 165 | 0.059 | 10 | 18.5 | 1.076 | 0.827 | 1.09 | 47.5 |
| 10 | 165 | 0.094 | 10 | 21.5 | 1.495 | 1.005 | 0.96 | 43.9 |
| 11 | 165 | 0.200 | 10 | 25 | 2.691 | 1.401 | 0.77 | 37.5 |
| 12 | 165 | 0.028 | −10 | 12.5 | 0.805 | 0.881 | 0.77 | 37.6 |
| 13 | 165 | 0.059 | −10 | 16 | 1.312 | 1.294 | 0.69 | 34.6 |
| 14 | 165 | 0.094 | −10 | 19 | 1.784 | 1.557 | 0.60 | 31.1 |
| 15 | 165 | 0.200 | −10 | 22.5 | 3.105 | 2.100 | 0.45 | 24.1 |

TABLE 2

Shear Angle $\Phi_0$ and Average Friction Angle $\beta_0$ Determined with the Passive Force Taken into Account

| Experiment number | $a$, mm | $l$, mm | $\alpha$, deg | $\beta$, deg | $\Phi$, deg | $\Phi_1$, deg | $\Phi_2$, deg | $\beta_0$, deg | $\Phi_0$, deg |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.094 | 0.321 | 10 | 46.4 | 17 | 26.8 | 20.3 | 43.3 | 20.3 |
| 2 | 0.094 | 0.289 | 10 | 48.2 | 19 | 25.9 | 19.4 | 45.6 | 19.2 |
| 3 | 0.094 | 0.256 | 10 | 43.4 | 21.5 | 28.3 | 21.8 | 39.9 | 22.0 |
| 4 | 0.094 | 0.222 | 10 | 39.0 | 25 | 30.5 | 24.0 | 34.8 | 24.6 |
| 5 | 0.094 | 0.085 | −10 | 32.8 | 16.5 | 23.6 | 17.1 | 31.0 | 16.5 |
| 6 | 0.094 | 0.187 | −10 | 30.3 | 19 | 24.8 | 18.3 | 28.3 | 17.9 |
| 7 | 0.094 | 0.256 | −10 | 26.5 | 22 | 26.8 | 20.3 | 24.0 | 20.0 |
| 8 | 0.028 | 0.474 | 10 | 48.5 | 19 | 25.8 | 19.3 | 38.8 | 22.6 |
| 9 | 0.059 | 0.331 | 10 | 47.5 | 18.5 | 26.2 | 19.7 | 43.1 | 20.4 |
| 10 | 0.094 | 0.289 | 10 | 43.9 | 21.5 | 28.0 | 21.5 | 40.7 | 21.7 |
| 11 | 0.200 | 0.251 | 10 | 37.5 | 25 | 31.2 | 24.7 | 35.4 | 24.3 |
| 12 | 0.028 | 0.128 | −10 | 37.6 | 12.5 | 21.2 | 14.7 | 33.4 | 15.3 |
| 13 | 0.059 | 0.216 | −10 | 34.6 | 16 | 22.7 | 16.2 | 32.3 | 15.9 |
| 14 | 0.094 | 0.289 | −10 | 31.1 | 19 | 24.4 | 17.9 | 29.3 | 17.4 |
| 15 | 0.200 | 0.523 | −10 | 24.1 | 22.5 | 28.0 | 21.5 | 22.7 | 20.6 |

calculated values of $\Phi_1$, $\Phi_2$, and $\Phi_0$: the value of $\Phi_1$ is determined by formula (1), $\Phi_2$ by formula (2) for $\varphi = 13°$ and $F_0 = 0$, and the value of $\Phi_0$ is determined by formula (2) for $\varphi = 16°$ taking into account the passive force. The results given in Table 2 show that accounting for the resistance force at the cutting edge provides good agreement between results of experiments and calculations using formula (2) for the shear angle and does not change the main conclusion of [3].

**4. Effect of the Average Normal Stress on Plasticity.** A large number of critical remarks [5–9] refers to Merchant's assumption [3] that, on the shear plane, the shear stress depends on the size of the normal stress. Merchant believed that this assumption is based on Bridgman's work [15], but it has previously been noted that the results of [3], in particular formula (2), can be obtained using the Mohr–Coulomb criterion [16]. We show that for the internal friction angles obtained in [3], the results of the solution using the Mohr–Coulomb criterion agree well with the results of experiments on steel samples under the joint action of tension $\sigma = \sigma_x$ and torsion $\tau = \tau_{xz}$ [17]. In this case, the principal stresses are determined by the formulas

$$\sigma_1 = \sigma/2 + \sqrt{\sigma^2 + 4\tau^2}/2, \qquad \sigma_2 = 0, \qquad \sigma_3 = \sigma/2 - \sqrt{\sigma^2 + 4\tau^2}/2.$$

Substituting the above stress values into (3) and performing some transformations, we obtain the Coulomb–Mohr ellipse

$$\Big(\frac{\sigma + 2C\tan\varphi}{2C/\cos\varphi}\Big)^2 + \Big(\frac{\tau}{C}\Big)^2 = 1, \tag{13}$$

where $C = (1 + \sin\varphi)\sigma_t/(2\cos\varphi)$ and $\sigma_t$ the yield limit for uniaxial tension.

Figure 6 gives curves of the stresses $\tau/\sigma_t$ versus $\sigma/\sigma_t$ on the yield surface. It is evident that the results of experiments with steel samples are the closest to the results obtained using the Mohr–Coulomb criterion (13).

**5. Conclusions.** The aforesaid leads to the following conclusions.

The solution proposed in the present paper for the shear angle (7), which is a generalization of the Lee–Shaffer solution, is in satisfactory agreement with experimental results [4].

Accounting for the resistance force at the cutting edge provides satisfactory agreement between the results of experiments and calculations using formula (2) for the shear angle and does not change the main conclusion of [3].

For close cutting parameters (cutting rate and the frontal angle of the cutter), experimental data [3, 4] are in good agreement with the results of solution (2).

For the internal friction angles obtained in [3], the results of calculations using the Mohr–Coulomb criterion agree well with the results of experiments with steel samples under the joint action of tension and torsion [17] and can be used in metal cutting problems.

This work was supported by the Russian Foundation for Basic Research (Grant No. 08-08-00113).

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